

# Linear State Space Modeling of Gamma-Ray Burst Lightcurves

David Band<sup>1</sup>, Michael König<sup>2</sup> and Anton Chernenko<sup>3</sup>

<sup>1</sup>*CASS, UC San Diego, La Jolla, CA 92093*

<sup>2</sup>*University of Tübingen, Germany*

<sup>3</sup>*Space Research Institute, Moscow, Russia*

**Abstract.** Linear State Space Modeling determines the hidden autoregressive (AR) process in a noisy time series; for an AR process the time series' current value is the sum of current stochastic “noise” and a linear combination of previous values. We present preliminary results from modeling a sample of 4 channel BATSE LAD lightcurves. We determine the order of the AR process necessary to model the bursts. The comparison of decay constants for different energy bands shows that structure decays more rapidly at high energy. The resulting models can be interpreted physically; for example, they may reveal the response of the burst emission region to the injection of energy.

## INTRODUCTION

Hidden in BATSE's superb gamma-ray burst lightcurves in different energy bands are temporal and spectral signatures of the fundamental physical processes which produced the observed emission. Various techniques have been applied to the BATSE data to extract these signatures, such as: auto- and crosscorrelations of lightcurves in different energies [1]; Fourier transforms [2]; lightcurve averaging [3]; Cross-Fourier transforms [4] and pulse fitting [5]. Here we propose to use Linear State Space Models (LSSM) to study the gamma-ray burst lightcurves.

LSSM estimates a time series' underlying autoregressive (AR) process in the presence of observational noise. An AR process assumes that the real time series is a linear function of its past values (“autoregression”) in addition to “noise,” a stochastic component of the process. Since the noise adds information to the system, it is sometimes called the “innovation” [6]. A moving average of the previous noise terms is equivalent to autoregression, and therefore these models are often called ARMA (AutoRegressive, Moving Average) processes [6]. While ARMA processes are simply mathematical models of a time series, the resulting model can be interpreted physically, which is the purpose of their application to astrophysical systems. For example, the noise may be the injection of energy into an emission

region, while the autoregression may be the response of the emission region to this energy injection, such as exponential cooling.

The application of LSSM to burst lightcurves can be viewed as an exploration of burst phenomenology devoid of physical content: how complicated an AR process is necessary to model burst lightcurves? Can all bursts be modeled with the same AR process? However, because different types of AR processes can be interpreted as the response of a system to a stochastic excitation, characterizing bursts in terms of AR processes has physical implications. Since we have lightcurves in different energy bands, we can compare the response at different energies. For example, the single coefficient in the AR[1] process (the nomenclature is described below) is a function of an exponential decay constant. If the lightcurves in all energy bands can be modeled by AR[1] then we have decay constants for every energy band. Since most bursts undergo hard-to-soft spectral evolution [7,1] and temporal structure is narrower at high energy than at low energy [8], we expect the decay constants to be shorter for the high energy bands.

## LINEAR STATE SPACE MODELS

The purpose of the LSSM methodology is to recover the hidden AR process. If the time series  $x(t)$  is an AR[p] process then

$$x(t) = \sum_{i=1}^p a_i x(t-i) + \epsilon(t, \sigma_x^2) \quad (1)$$

where time is assumed to advance in integral units. The “noise” (or “innovation”)  $\epsilon(t, \sigma_x^2)$  is uncorrelated and possesses a well-defined variance  $\sigma_x^2$ ; the noise is usually assumed to be Gaussian. Since the burst count rate cannot be negative, we expect the noise also cannot be negative. A Kolmogorov-Smirnov test is used to determine when p is large enough to model the system adequately [9,10].

If p=1, the system responds exponentially to the noise with a decay constant  $\tau$ , and

$$a_1 = e^{-1/\tau} \quad . \quad (2)$$

The p=2 system is a damped oscillator with period  $T$  and relaxation time  $\tau$ ,

$$a_1 = 2 \cos\left(\frac{2\pi}{T}\right) e^{-1/\tau} \quad \text{and} \quad a_2 = e^{-2/\tau} \quad . \quad (3)$$

Thus, the lowest order AR processes lend themselves to obvious physical interpretations.

Unfortunately, we do not detect  $x(t)$  directly, but a quantity  $y(t)$  which is a linear function of  $x(t)$  and observational noise:

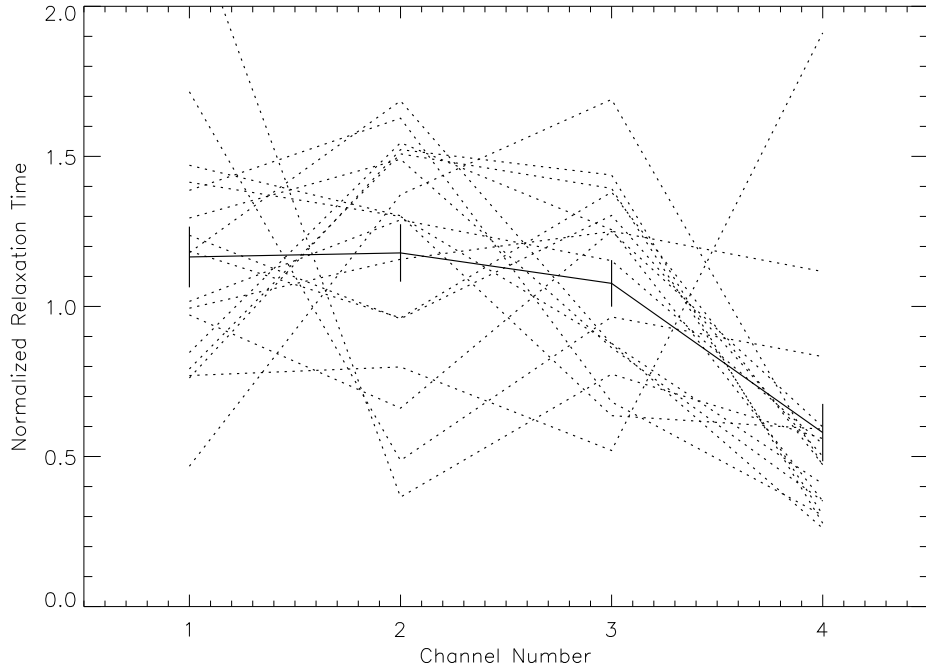
$$y(t) = Cx(t) + \eta(t, \sigma_y^2) \quad (4)$$

where in our case  $C$  is an irrelevant multiplicative factor and  $\eta$  is a zero-mean noise term with variance  $\sigma_y^2$ ;  $\eta$  is also often assumed to be Gaussian. The LSSM code uses the Expectation-Maximization algorithm [9,10].

## APPLICATION TO BURSTS

We have thus far applied our LSSM code [10] to 17 gamma-ray bursts. We used the 4-channel BATSE LAD discriminator lightcurves extracted from the DISCSC, PREB, and DISCLA datatypes, which have 64 ms resolution; the energy ranges are 25–50, 50–100, 100–300 and 300–2000 keV. Each channel was treated separately, resulting in 68 lightcurves. Of these lightcurves, 52 could be modeled by AR[1], 13 by AR[2] and 3 by AR[4]. Thus there is a preference for the simplest model, AR[1]. Note that Chernenko *et al.* [11] found an exponential response to a source function in their soft component.

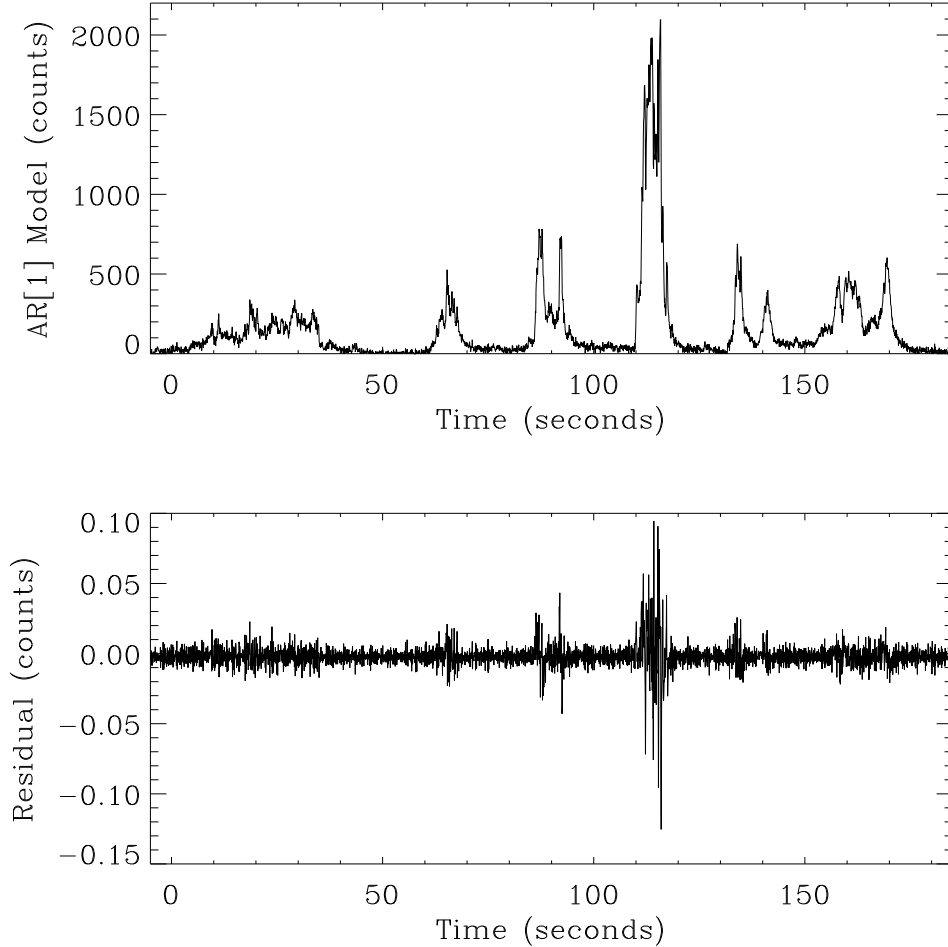
Figure 1 presents the normalized relaxation time constants for the bursts in our sample, as well as their average. Even for models more complicated than AR[1] a relaxation time constant can be identified. As expected, the averages of these time constants become shorter as the energy increases from channel 1 to channel 4,



**FIGURE 1.** AR[1] time constants by channel, normalized by the average over all channels. The dotted curves are for each of the 17 bursts, while the solid curve is the average. Note that the time constants decrease from channel 1 to channel 4, as expected for hard-to-soft evolution, although there is a great deal of scatter for the individual bursts.

consistent with the trend found in quantitative studies of spectral evolution [1,7] and the qualitative inspection of burst lightcurves.

In Figure 2 we present the analysis of GRB 940217, the burst with an 18 GeV photon 90 minutes after the lower energy gamma-ray emission ended [12]. As can be seen, the residuals are much smaller than the model and are consistent with fluctuations around 0; plots for the data and the model are indistinguishable, and only one is presented. The amplitude of the residuals increases as the count rate increases (attributable in part to counting statistics), but there is no net deviation from 0.



**FIGURE 2.** AR[1] model for channel 2 (50-100 keV) of GRB 940217 (top) and the residuals ( $\eta$  in eq. [4]) between the data and the model (bottom).

## FUTURE DIRECTIONS

We plan to apply the LSSM code to a large number of bursts. We will compare the order of the underlying AR process and the resulting coefficients obtained for the different energy lightcurves of the same burst and for different bursts. In this way we can search for hidden classes of bursts and explore the universality of the physical processes.

The “noise”  $\epsilon(t)$  might be a measure of the energy supplied to the emission region (although which physical processes are the noise and which the response is model dependent). Therefore characterizing  $\epsilon(t)$  may probe a deeper level of the burst phenomenon. The  $\epsilon(t)$  lightcurves for the different energy bands should be related; we expect major events to occur at the same time in all the energy bands, although the relative intensities may differ.

Many of the bursts consist of well-separated spikes or complexes of spikes. We will apply the LSSM code to each part of the burst to determine whether the same order AR process characterizes the entire burst, and if so, whether the AR process has the same coefficients. This will test whether the physical processes remain the same during the burst.

## ACKNOWLEDGMENTS

D. Band’s gamma-ray burst research is supported by the *CGRO* guest investigator program and NASA contract NAS8-36081.

## REFERENCES

1. Band, D. L., *Ap. J.* **486**, 928 (1997).
2. Shaviv, N., Ph.D. Thesis, Technion (1996).
3. Mitrofanov, I., *et al.*, *Ap. J.* **459**, 570 (1996).
4. Kouveliotou, C., *et al.*, in *Gamma-Ray Bursts, AIP Conf. Proc. 265*, eds. W. Paciesas and G. Fishman (New York: AIP), 299 (1992).
5. Norris, J. P., *et al.*, *Ap. J.* **459**, 393 (1996).
6. Scargle, J. D., *Ap. J. Suppl.* **45**, 1 (1981).
7. Ford, L. A., *et al.*, *Ap. J.* **439**, 307 (1995).
8. Fenimore, E. E., *et al.*, *Ap. J. Lett.* **448**, L101 (1995).
9. Honerkamp, J., *Stochastic Dynamical Systems*, (New York: VCH Publ.) (1993).
10. König, M., and Timmer, J., *Astron. Astrophys.* **124**, 589 (1997).
11. Chernenko, A., *et al.*, these proceedings (1998).
12. Hurley, K., *et al.*, *Nature* **372**, 652 (1994).